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A Derivation for the Noise Temperature of a Horn-type Noise Standard

W. C. Daywitt

National Bureau of Standards, Boulder, CO 80303, USA

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Abstract

Noise sources consisting of an electromagnetic horn aimed at an absorbing material have been in use for many years. A satisfactory derivation of the noise temperature for such a configuration has been missing, however, preventing the use of this horn-type noise source as a primary reference standard. The derivation described in this paper models the various noise emitters within the source well enough to provide an accurate estimate of the noise temperature and a complete error analysis.

1. Introduction

For the past twenty years the Electromagnetic Fields Division of the U.S. National Bureau of Standards has built a number of coaxial and waveguide noise sources [1–3] consisting of single-moded uniform transmission lines terminated with reflectionless loads. The accuracy of their calculated noise temperatures is typically 1%, and tends to degrade as the operating frequency increases. The basic design is illustrated in Fig. 1 where the termination and a portion of the transmission line are immersed in a thermal reservoir at temperature T_m, with the remaining portion of the line leading to the output connector at room temperature T_0. The temperature distribution T_x of the line is also illustrated, where the room-temperature portion of the line has a length l. Radiation from the termination and the dissipative losses in the line result in the noise temperature [4]

$$T_n = T_m + \Delta T \tag{1}$$

where, for the simple distribution shown,

$$\Delta T \doteq (2\,a\,l)\,(T_0 - T_m) \tag{2}$$

and the attenutation coefficient a refers to the line at temperature T_0. The equations indicate that only the portion of the line at T_0 contributes to the excess (in excess of thermal equilibrium conditions) noise temperature ΔT.

The largest source of error in calculating the noise temperature by (1) is the attenutation $2al$ which is usually estimated with an uncertainty varying from 10% to 20%. With this large an uncertainty, it is necessary to keep the attenuation small (to maintain the uncertainty in T_n less than 1%) implying either a small attenuation coefficient a, a short transition length l, or both. In the microwave frequency range and below, the attenuation can be kept down with relatively simple engineering designs, but this becomes more of a problem as the frequency increases into the millimeter-wave range. To circumvent the engineering difficulties the transmission-line type of noise standard was abandoned in favor of a design [5] incorporating a millimeter-

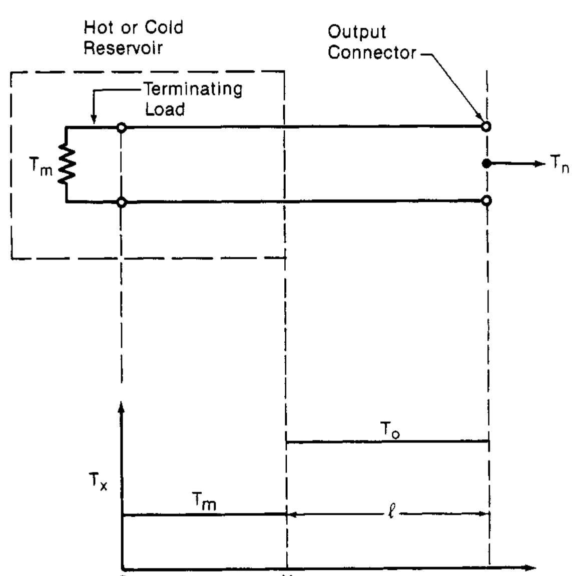

Fig. 1. A transmission-line type noise standard

wave horn antenna "looking" at an absorber of known temperature. The resulting antenna noise temperature [4] is close to the measured temperature T_m of the absorber, with additional noise contributions from the dissipative antenna losses and from the antenna side and back lobes. This type of noise source is not new, but its use as a primary reference standard required the successful resolution of two previously unresolved problems: the need of a useful expression for the noise efficiency (defined later) of the antenna with sources in its radiating near field [6]; and an estimate of the error caused by near-field excess radiation entering the side and back lobes of the antenna.

Figure 2 shows a schematic diagram of the cryogenic, millimeter-wave noise standard [5] constructed to overcome the engineering difficulties just mentioned. It consists of an electromagnetic horn at room temperature T_0 with its waveguide lead protruding from the enclosure whose conducting walls vary in temperature from T_0 near the horn to the absorber temperature T_m (77 K). The noise temperature T_n of the standard can be expressed as the sum of four terms:

$$T_n = T_m + \Delta T_h + \Delta T_c + \Delta T_i \,. \tag{3}$$

The first term, equal in magnitude to the absorber temperaure, is the noise temperature of the standard when the walls and horn are at the absorber temperature, and includes emission from the entire cavity (horn, conducting walls and absorber). The second term is the excess noise temperature contribution from the horn due to its elevated temperature ($T_0 > T_m$), and is calculated as though the horn were in a reflectionless cavity at the absorber temperature. The third term is the excess noise from the conducting walls, neglecting multiple reflections between the walls and the horn. The last excess noise temperature is an interaction term which includes the effects on the horn and wall contributions from multiple reflections between the horn and walls, and the error due to assuming that the horn is within a reflectionless cavity while deriving the second term.

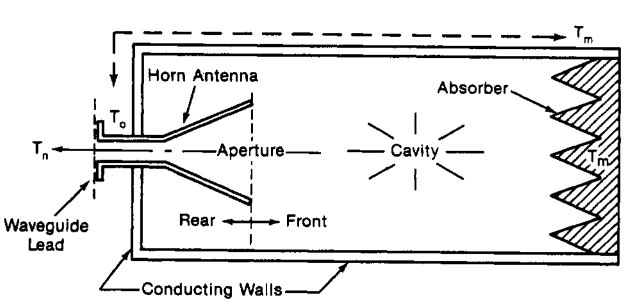

Fig. 2. A horn-type noise standard

These three excess noise temperatures vanish if the horn and walls are at the absorber temperature, or if the horn and walls are infinitely conducting. The purpose in writing the noise temperature in the form of (3) is twofold: for a horn of moderate gain the last two terms are small and can be discarded, requiring only an upper-bound estimate of their magnitudes; and an expression for the second term can be derived from rigorous, thermodynamical arguments even with the cavity in the radiating near field of the antenna. This second term is expressed in the form

$$\Delta T_h = (1-\alpha)\,(T_0 - T_m) \tag{4}$$

where α is the noise efficiency, and accounts for both absorption and emission by dissipative loss in the horn structure. The same expression holds for a lossy, two-port, microwave junction [7], where the noise efficiency reduces to an available power ratio, and leads to the approximation in (2).

2. Antenna Noise Efficiency

The following antenna-noise efficiency derivation employs the plane-wave scattering matrix theory of antennas [8] and is in two steps. First, a spectral-density function for the cavity radiation is found by placing an arbitrary, lossless and reciprocal antenna in the cavity, and applying the scond law of thermodynamics. Then, the lossless antenna is replaced by a *lossy*, possibly different, antenna and the density function and the second law of thermodynamics are used again to calculate the noise efficiency for the lossy antenna.

Figure 3 shows a lossless antenna terminated by a reflectionless load in thermal equilibrium with the cavity via radiation through the antenna, leaving the load and cavity at the same temperature T_m. The cavity walls are infinite in extent, and lie entirely to the left or right of the planes denoted by "q = 1" or "q = 2". $da_q(m, \boldsymbol{K}, \boldsymbol{r})$ is a differential amplitude function for the radiation originating at position $\boldsymbol{r}$, with q denoting the front and back hemispheres surrounding the antenna. m is the polarization index of the corresponding wave incident on the antenna, and $\boldsymbol{K}$ is the transverse (to the z-direction) part of the propagation vector $\boldsymbol{k}$. The wave amplitude of the field in the antenna waveguide lead produced by da_q is represented by db_0, and S_{00} is the antenna reflection coefficient. Since there are no multiple reflections between the antenna and the cavity walls [8]

$$db_0 = \sum\!\!\!\!\!\!\int S_{0q}(m, \boldsymbol{K})\, da_q(m, \boldsymbol{K}, \boldsymbol{r})\, \mathbf{d}\boldsymbol{K} \tag{5}$$

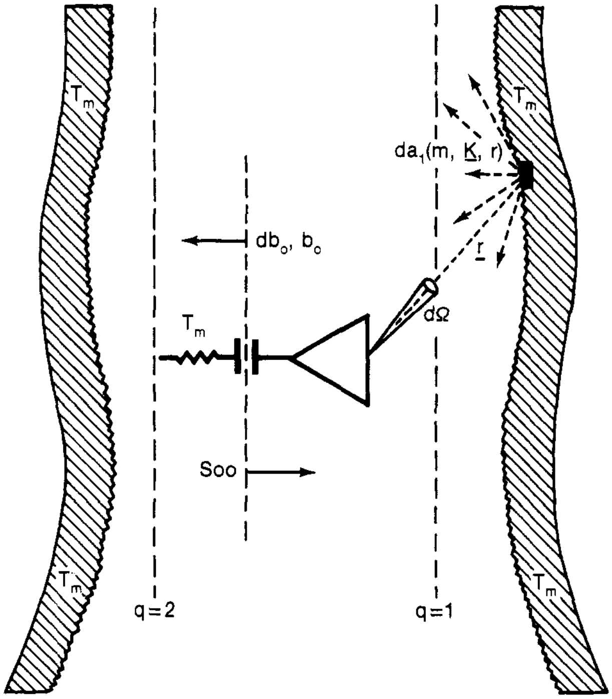

Fig. 3. An antenna and waveguide termination in a reflectionless, isothermal cavity

where the symbol

$$\not{\int} \equiv \sum_{q} \int_{K} \sum_{m} \tag{6}$$

is used for convenience, and where the integral is two-dimensional, being performed over the product $dk_x\, dk_y$ symbolized by $\mathbf{d}\boldsymbol{K}$. The integral limits are restricted to the region in $\boldsymbol{k}$-space where $K < k$, implying that there is no evanescent-mode coupling between the antenna and cavity. S_{0q} is the receiving characteristic of the antenna. The total wave amplitude from all of the sources in the cavity walls is

$$b_0 = \int_{4\pi} db_0 = \not{\int} S_{0q}(\mathrm{m},\boldsymbol{K}) \int_{2\pi} da_q(\mathrm{m},\boldsymbol{K},\boldsymbol{r})\, \mathbf{d}\boldsymbol{K} \tag{7}$$

where the integral is over the front (q = 1) or back (q = 2) hemisphere.

The spectral density ω_0 of the available power corresponding to b_0 for a unit bandwidth is

$$\omega_0 = \frac{\eta_0}{1 - |S_{00}|^2} \langle b_0 b_0^* \rangle \tag{8}$$

where the bracket symbolizes an ensemble average, accounting for the fact that b_0 is a stochastic variable [9], and where the asterisk stands for the complex conjugate. η_0 is the characteristic admittance for the mode propagating in the waveguide. Combining (7) and (8) leads to

$$\omega_0 = \frac{\eta_0}{1 - |S_{00}|^2} \not{\int}\not{\int}' S_{0q}(\mathrm{m},\boldsymbol{K})\, S_{0q'}^*(\mathrm{m}',\boldsymbol{K}')\, \omega_c\, \mathbf{d}\boldsymbol{K}\, \mathbf{d}\boldsymbol{K}' \tag{9}$$

where

$$\omega_c \equiv \int_{2\pi} \int_{2\pi} \langle da_q(\mathrm{m},\boldsymbol{K},\boldsymbol{r})\, da_{q'}^*(\mathrm{m}',\boldsymbol{K}',\boldsymbol{r}') \rangle\,. \tag{10}$$

The power spectral density ω of the load attached to the antenna is [9]

$$\omega = \varkappa T_m \tag{11}$$

where $\varkappa$ is Boltzmann's constant, and T_m is the load temperature. Since the load and cavity are in thermal equilibrium, the second law of thermodynamics [10] insures that

$$\omega_0 = \omega\,. \tag{12}$$

Combining (9), (11) and (12) leads to

$$\frac{\eta_0}{1 - |S_{00}|^2} \not{\int}\not{\int}' S_{0q}(\mathrm{m},\boldsymbol{K})\, S_{0q'}^*(\mathrm{m}',\boldsymbol{K}')\, \omega_c\, \mathbf{d}\boldsymbol{K}\, \mathbf{d}\boldsymbol{K}' = \varkappa T_m\,. \tag{13}$$

Since the antenna is lossless and reciprocal [8]

$$\frac{\eta_0}{1 - |S_{00}|^2} \not{\int} \frac{|S_{0q}(\mathrm{m},\boldsymbol{K})|^2}{\eta_m(K)}\, \mathbf{d}\boldsymbol{K} = 1 \tag{14}$$

where $\eta_m(K)$ is the wave admittance for the incident wave with polarization index m and transverse wave number K. $\delta_{qq'}$ and $\delta_{mm'}$ are Kronecker delta functions, and $\delta(\boldsymbol{K}-\boldsymbol{K}')$ is a Dirac delta function. Multiplying (14) by $\varkappa T_m$ and combining it with (13) leads to

$$\frac{\eta_0}{1 - |S_{00}|^2} \not{\int}\not{\int}' S_{0q}(\mathrm{m},\boldsymbol{K})\, S_{0q'}^*(\mathrm{m}',\boldsymbol{K}') \left[\omega_c - \frac{\varkappa T_m}{\eta_m(K)} \delta_{qq'} \delta_{mm'} \delta(\boldsymbol{K}-\boldsymbol{K}') \right] \mathbf{d}\boldsymbol{K}\, \mathbf{d}\boldsymbol{K}' = 0\,. \tag{15}$$

Since (15) must hold for any lossless, reciprocal antenna, and since ω_c is independent of the antenna parameters, the quantity in the bracket must vanish. This yields the required spectral function for the cavity radiation,

$$\omega_c = \frac{\varkappa T_m}{\eta_m(K)} \delta_{qq'} \delta_{mm'} \delta(\boldsymbol{K}-\boldsymbol{K}')\,. \tag{16}$$

The lossless antenna is now replaced by a lossy antenna, the resulting noise temperature T_n being equal to the power spectral density (divided by Boltzmann's constant) of the total available noise power at the antenna waveguide flange. Since the total power is the sum of the independent noise powers generated by the cavity and the dissipative losses in the antenna, the spectral density is the sum of the separate spectral densities of the antenna and cavity. The spectral density ω' for the cavity radiation appearing at the waveguide flange of the antenna can be obtained by inserting (16) into (9) and taking the cavity temperature T_m (since it is constant) out from under the integral sign. The result is

$$\omega' = \varkappa T_m \alpha \tag{17}$$

where the noise efficiency α turns out to be

$$\begin{aligned}\alpha &= \frac{\eta_0}{1-|S'_{00}|^2} \sum\!\!\!\!\!\!\int \frac{|S'_{0q}(\mathrm{m},\boldsymbol{K})|^2}{\eta_m(K)}\,\mathrm{d}\boldsymbol{K} \\ &= \frac{\eta_0 k}{(1-|S'_{00}|^2)\,Y_0} \sum\!\!\!\!\!\!\int \frac{|s'_{0q}(\boldsymbol{K})|^2}{\gamma}\,\mathrm{d}\boldsymbol{K}.\end{aligned} \tag{18}$$

The second sum-integral symbol includes only the sum over q as the sum over m is absorbed into the "complete" antenna receiving characteristic s'_{0q} [8]. Y_0 is the wave admittance of free space, *and the primes in the equation signify that the corresponding parameters belong to a lossy antenna.* For the reciprocal antenna (18) reduces to the antenna radiation efficiency [8].

It proves useful to rearrange (18) into

$$\alpha = \sum\!\!\!\!\!\!\int \eta_{0q}(\boldsymbol{K})\,\mathrm{d}\alpha \tag{19}$$

where

$$\eta_{0q}(\boldsymbol{K}) \equiv \frac{1-|S_{00}|^2}{1-|S'_{00}|^2}\left|\frac{s'_{0q}(\boldsymbol{K})}{s_{0q}(\boldsymbol{K})}\right|^2, \tag{20}$$

$$\mathrm{d}\alpha \equiv \frac{\eta_0 k\,|s_{0q}(\boldsymbol{K})|^2}{Y_0\gamma(1-|S_{00}|^2)}\,\mathrm{d}\boldsymbol{K} \tag{21}$$

and γ is the z-component of the propagation vector $\boldsymbol{k}$. The unprimed quantities refer to the lossless antenna which is an exact duplicate of the lossy antenna except for infinite conductivity. The antenna loss is now contained entirely in (20), and

$$\sum\!\!\!\!\!\!\int \mathrm{d}\alpha = 1. \tag{22}$$

Since the antenna is reciprocal [8]

$$\eta_0 k\, s_{0q}(\boldsymbol{K}) = Y_0\gamma\, s_{q0}(-\boldsymbol{K}). \tag{23}$$

A similar expression holds for the lossy or primed quantities, and s_{q0} is the "complete" transmitting characteristic of the antenna. Equations (20) and (21) can now be expressed in terms of the transmitting characteristic

$$\eta_{0q}(\boldsymbol{K}) = \frac{1-|S_{00}|^2}{1-|S'_{00}|^2}\left|\frac{\gamma s'_{q0}(-\boldsymbol{K})}{\gamma s_{q0}(-\boldsymbol{K})}\right|^2 \tag{24}$$

and

$$\mathrm{d}\alpha = \frac{Y_0|\gamma s_{q0}(-\boldsymbol{K})|^2}{\eta_0(1-|S_{00}|^2)}\frac{\mathrm{d}\boldsymbol{K}}{k\gamma}. \tag{25}$$

Equations (24) and (25) depend only upon the antenna structure, and apply just as well whether the noise sources in the cavity walls are in the radiating near field or the far field of the antenna. Therefore (24) and (25) may be evaluated in terms of the transmitted far field [8] leading to

$$\eta_{0q}(\boldsymbol{K}) = \frac{1-|S_{00}|^2}{1-|S'_{00}|^2}\left|\frac{r\boldsymbol{E}'_q(\boldsymbol{r})}{r\boldsymbol{E}_q(\boldsymbol{r})}\right|^2 \tag{26}$$

and

$$\mathrm{d}\alpha = \frac{P_n(\mathrm{q},\boldsymbol{K})\,\mathrm{d}\Omega}{\Omega_a} \tag{27}$$

$r\boldsymbol{E}_q$ is the far-field pattern. P_n and Ω_a are the normalized power pattern and antenna solid angle respectively, and $\mathrm{d}\Omega$ is the differential solid angle referred to the antenna aperture. Combining (19), (26) and (27),

$$\alpha = \frac{1-|S_{00}|^2}{1-|S'_{00}|^2}\frac{1}{\Omega_a}\int\left|\frac{r\boldsymbol{E}'(\boldsymbol{r})}{r\boldsymbol{E}(\boldsymbol{r})}\right|^2 P_n(\theta,\varphi)\,\mathrm{d}\Omega \tag{28}$$

where θ and φ are the usual antenna angles, and where S'_{00} and the field pattern $r\boldsymbol{E}'(\boldsymbol{r})$ contain the antenna loss. The integral is over the entire 4π solid angle surrounding the antenna, and the ratio in the integrand is the ratio of the field pattern with loss to the field pattern without loss.

Equation (28) is the desired noise efficiency in a form that permits the components of the horn loss to be traced [5] from the waveguide lead of the horn, out through the flare and aperture, to the external surfaces of the structure. Furthermore, since most of the horn loss is generated in its waveguide and throat areas, and since S'_{00} is very nearly equal to S_{00}, the right-hand side of (28) can be easily and accurately estimated [5].

3. Excess Cavity Noise

The noise standard was constructed in such a way that multiple reflections between the horn aperture and the cavity walls can only take place between the aperture and the rear walls (see Fig. 2). Thus, for a horn with sufficient gain the excess noise temperature ΔT_c due to the cavity walls can be estimated without including these reflections. Considering just the E-plane of the horn, Fig. 4 depicts a small radiating source of area $\mathrm{d}A$ in the absorber and its reflections generated by the conducting walls of the cavity, and resulting in the noise temperature $\mathrm{d}T_n$. For simplicity, the apparent positions of these images (the squares in the figure) are calculated assuming the horn to be missing from the cavity, an assumption that does not have a significant effect on the results. As seen from the H-edges of the horn (points 1 and 2) the more distant images look closer together, appearing to condense on the "condensation line" shown. Since the magnetic source currents in the E-plane of the figure produce electric fields

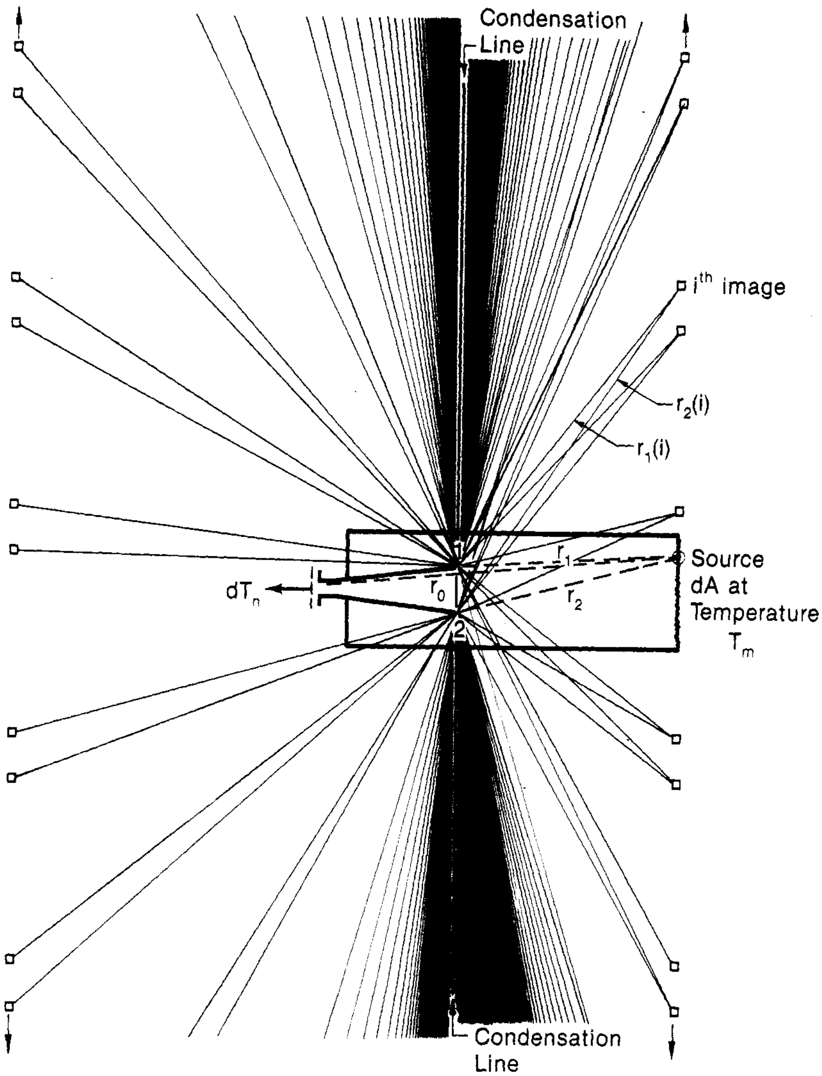

Fig. 4. A source and its images in the E-plane of the horn

that are cross-polarized to the antenna and thus not received, only currents perpendicular to the figure need be considered and these can be easily accounted for by summing their corresponding magnetic fields in the horn throat. To first order this sum takes the form [11]

$$H = H_0 \frac{e^{-jkr_0}}{kr_0} + H_1(0)\frac{e^{-jkr_1}}{kr_1}(1,0) + H_2(0)\frac{e^{-jkr_2}}{kr_2}(2,0) + \sum_1^{\infty} Z_{1i} H_1(i)\frac{e^{-jkr_1(i)}}{kr_1(i)}(1,i) + \sum_1^{\infty} Z_{2i} H_2(i)\frac{e^{-jkr_2(i)}}{kr_2(i)}(2,i)\,. \quad (29)$$

The first term is the magnetic field at the throat that comes directly from the source, where H_0 is the source strength, r_0 is the radius from the throat to the source, and k is the free-space wavenumber. This is the only component that reaches the throat directly (the images must diffract around the aperture edges at points 1 and 2 to reach throat) because only the absorber can be seen from the throat area. The second term is due to diffraction of the source into the throat by edge 1, where $(1,0)$ is the diffraction coefficient, and r_1 is the radius from the source to this edge. The source strength $H_1(0)$ differs from H_0 since the angles at which the radiation is emitted from the surface area dA are different in the two cases. The third term has an interpretation similar to the second term. The two sums in the equation are the contributions from the images, where identification of the last three factors in each sum is evident from the previous discussion. The two factors Z_{1i} and Z_{2i} are reflection coefficients that account for the loss arising from the wall reflections that give rise to the images.

The noise temperature resulting from the radiation of dA is calculated from

$$dT_n = C\langle HH^*\rangle \quad (30)$$

where C is a constant relating the throat field H to the TE_{10} mode field in the waveguide lead of the horn, and where the second factor represents an ensemble average of the magnetic field times its complex conjugate. When (29) and (30) are combined, there emerges an equation of the form

$$\mathrm{d}T_n = C' T\{\}\,\mathrm{d}A \tag{31}$$

where C' is another constant which includes C. The temperature T appears in the equation because the ensemble average is proportional to the temperature of $\mathrm{d}A$. The proportionality "constant" is symbolized by the bracket and depends upon the temperature distribution of the metallic walls since the wall conductivity is a function of the temperature. Equation (31) equally well describes radiation from a source in the conducting walls of Fig. 4, the only difference being that the first term in (29) for the throat field is missing because the walls cannot be seen from the throat.

The influence of radiation from a source in the conducting walls on the noise temperature relative to a source in the absorber is greater for the E-plane shown in Fig. 4 than for any other plane through the axis of the horn. Therefore, if (31) is used for describing the radiation from other planes, the effect of the wall radiation on the noise temperature is over-emphasized, leading to an overestimate of the excess noise temperature ΔT_c. Since this term in (3) is to be discarded, and since a calculation similar to the preceding one in these other planes is prohibitively difficult, it is convenient and conservative to think of $\mathrm{d}A$ as representing the differential surface area of a cylindrical ring around the horn axis to account for these radiating sources in the conducting walls of the cavity. Equation (31) is then integrated to account for the total cavity radiation with the corresponding noise temperature

$$T_n = C' \int T\{\}\,\mathrm{d}A\,. \tag{32}$$

Although this equation is too approximate to be used for calculating the noise temperature T_n, it is adequate for obtaining an upper bound to the excess cavity noise. The constant C' is calculated by noting that in thermal equilibrium (where the temperature of the walls and the horn are the same temperature T_m as the absorber) the noise temperature is equal to T_m. After evaluating C' in this way, and subtracting T_m from (32) to get the excess cavity noise, there results

$$\Delta T_c = \frac{\int T\{\}\,\mathrm{d}A}{\int \{\mathrm{m}\}\,\mathrm{d}A} \tag{33}$$

where the "m" in the lower bracket indicates that it is to be calculated with the cavity walls at the absorber temperature. When this ratio was evaluated [5] by computer it yielded an upper-bound error of 0.1% relative to the noise temperature calculated from the first two terms of (3).

4. Multiple Reflections

Multiple reflections taking place between the horn aperture and the rear cavity wall modify both the horn and the cavity-wall contributions to the noise temperature of the standard. The magnitude of the error due to changes in the horn-generated noise can be found by determining what portion of the standards's reflection coefficient is the result of rear-wall reflections. When this was done [5] the error was found to be negligible.

An upper bound to the wall-contribution error is estimated in a way similar to the calculation of the excess noise temperature in Section 3 by including images from the fields diffracted back into the rear of the cavity by the edges 1 and 2 in Fig. 4. By computer calculation the error from neglecting these first-order reflections was shown to be less than 0.05% [5]. Contributions from higher-order reflections fall off rapidly in magnitude compared to this value.

5. Conclusions

The analysis presented in this paper forms the basis for estimating the noise temperature of a horn-type

Table 1. Uncertainties in the WR10 (75 GHz – 110 GHz) noise standard

Source of uncertainty	Source uncertainty	Resulting percentage uncertainty in T_n
1. Higher modes	–	0
2. Multiple reflections between horn and cavity	–	+0 −0.05
3. Elevated cavity-wall temperature	–	+0 −0.10
4. Uncertainty in T_m	±0.26 K	±0.34
5. Uncertainty in T_0	±2 K	±0.02
6. Uncertainties in α		
6.1. Neglecting losses beyond aperture	–	+0 −0.01
6.2. Using waveguide loss equation	–	±0.01
6.3. Dimensional uncertainties	±0.0025 cm	±0.06
6.4. Uncertainties in dc resistivity curve	±5%	±0.01
6.5. Uncertainties due to surface roughness	±5%	±0.06
Total uncertainty (linear sum)		+0.5% −0.7%

noise standard. Previously, justification for such a configuration was not available, restricting standard building to the transmission-line type of configurations shown in Fig. 1. The analysis indicates that as long as the horn has sufficient gain and is contained in a cavity with highly conducting side and rear walls, the errors discussed can be kept within tolerable limits. As a practical matter, the silicon carbide absorber in the actual cavity [5] is broadband, so that one may use the same cryostat for other waveguide bands, being careful to position the horn aperture a few free-space wavelengths or more away from the cavity walls and the absorber.

A number of other errors not associated with the derivation just presented are also important, and attention is given to the design of the standard to keep these additional errors small. Table 1 indicates [5] the relative sizes of the various uncertainties for the millimeter-wave noise standard constructed for the WR10 waveguide band.

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